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Revisiting $B^+ \to X(3872) + K^+$ in pQCD assigning to $X(3872)$ $2\,^3P_1$ charmonium

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Abstract. We revisit the $B^+ \to X(3872) + K^+$ in the pQCD approach assigning to $X(3872)$ a $2\,^3P_1$ charmonium state. In this theoretical framework all the phenomenological parameters in the wavefunctions and Sudakov factor are a priori fixed by fitting other experimental data; therefore, there hardly are any free parameters in the whole numerical computations. Our results are larger than the upper bound set by the BABAR measurements.

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1 Introduction

In 2003, BELLE announced the observation of $X(3872)$ in the $B^\pm \to K^\pm \pi^+\pi^- J/\psi$ decay channel. Its mass and width are respectively $m = 3872.0 \pm 0.6 \pm 0.5\,\text{MeV}$ and $\Gamma < 2.3\,\text{MeV}$ [1]. Later the CDF, D0 and BABAR Collaborations also confirmed the existence of $X(3872)$ [2–4]. Very recently, the BELLE Collaboration reported $\bar{D}^0 D^0 \pi^0$ decay for $X(3872)$. However, its mass is $3875.4 \pm 0.7^{+1.2}_{-2.0}\,\text{MeV}$ [5].

$X(3872)$ has attracted a great deal of interest of both theorists and experimentalists in high energy physics. Some authors suppose that $X(3872)$ is conventional charmonium [6–10]. Meanwhile, some other authors suggest that $X(3872)$ is a molecular state of $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ [11–19] or a multiquark state [20]. The quantum number of $X(3872)$ favors $J^{PC} = 1^{++}$ from the angular distribution analysis by the BELLE Collaboration [21]. Thus, to $X(3872)$ is only assigned a $2\,^3P_1$ state for the explanation of charmonium. However, the mass of $X(3872)$ is lower by about 70 MeV than that predicted by the potential model of the past. If we trust the potential model calculation for the charmonium mass spectrum, $X(3872)$ will not favor a $2\,^3P_1$ $c\bar{c}$ state. However, in fact large uncertainties are involved in the calculation of the potential model near and above the open charm threshold. Thus we cannot rule out the charmonium assignment to $X(3872)$. At present one needs to carry out further studies on the nature of $X(3872)$ and especially to clarify its structure.

B non-leptonic decay is one of the suitable places for the production of charmonium. In this work, we assign to $X(3872)$ the regular $2\,^3P_1$ $c\bar{c}$ state which is the radial excited state of $\chi_{c1}(1\,^3P_1)$ and calculate the branching ratio of $B^+ \to X(3872)K^+$ in the perturbative QCD (pQCD) approach.

In [22, 23], Braaten et al. estimated $\text{BR}(B^+ \to X(3872)K^+) = (0.07\text{–}1) \times 10^{-4}$ assuming $X(3872)$ to be a s-wave molecular state of $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$. In [9], the authors calculated the $B \to X(3872)K$ ratio considering $X(3872)$ as a $2\,^3P_1$ $c\bar{c}$ state in QCD factorization, where the $2\,^3P_1$ $c\bar{c}$ state is described by a nonrelativistic wavefunction. They obtained $\text{BR}(B^0 \to X(3872)K^0) = \text{BR}(B^+ \to X(3872)K^+) \approx 2 \times 10^{-4}$. However, in a QCD improved factorization approach, the infrared divergence appearing in hard spectator correction diagrams can only be parameterized in a simple way: $X_\text{H} = \int_0^1 \frac{\mathrm{d}y}{1-y} = (1 + \varrho_\text{H} e^{\mathrm{i}\varphi_\text{H}}) \ln \frac{m_b}{\Lambda_h}$ [24, 25], which will result in a large uncertainty for the final result. Based on the above consideration and the only upper limit of $B^\pm \to X(3872)K^\pm$ as given by BABAR [26, 27], so assuming the $2\,^3P_1$ charmonium assignment for $X(3872)$, we revisit the $B^+ \to X(3872)K^+$ decay in the pQCD approach, which is believed to be successful for estimating the transition rates of B and D into light mesons [28–32] even though its applicability is under dispute [33].

Our numerical results indicate that the order of magnitude of $B^+ \to X(3872)K^+$ is $7.88^{+4.87}_{-3.76} \times 10^{-4}$, which is larger than that calculated in [9]. This difference probably for the most comes from the different models we adopt. The BABAR experiment only gave $\text{BR}(B^+ \to X(3872)K^+) < 3.2 \times 10^{-4}$ [26, 27]. On the assumption of $X(3872)$ being a pure $c\bar{c}$ state, we obtain the theoretical result that $\text{BR}(B^+ \to X(3872) + K^+)$ is larger than the upper bound given by BABAR within the error bar. Assuming that both our calculations in the pQCD approach and the experi-

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mental measurements of $B^+ \to X(3872) + K^+$ are reliable, $X(3872)$ cannot be simply categorized as a pure $c\bar{c}$ state, which is also entailed in [9]. However, due to the absence of a precise measurement of $\mathrm{BR}(B^+ \to X(3872) + K^+)$, which is only set by BABAR, we still wait for future experiments to confirm this point definitely. A more decisive conclusion should be made as more accurate data are accumulated by future experiments such as BELLE, BABAR and LHCb.

This paper is organized as follows. After this introduction, we formulate the decay amplitude of $B^+ \to X(3872)K^+$ in the pQCD approach, where $X(3872)$ is assigned a $2\,^3P_1$ $c\bar{c}$ state. Then we present our numerical results along with all the input parameters in Sect. 3. The last section is devoted to our conclusion and discussion. Some tedious expressions are collected in the appendix.

2 Formulation

In this work, we suppose that $X(3872)$ is a $2\,^3P_1$ charmonium state. The effective Hamiltonian relevant to $B^+ \to X(3872)K^+$ decay in the SM is written as [34]

$$\mathcal{H}_{\text{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}}\left[V_{cb}V_{cs}^*(\mathcal{C}_1(\mu)\mathcal{O}_1 + \mathcal{C}_2(\mu)\mathcal{O}_2) - V_{tb}V_{ts}^*\sum_{i=3}^{10}\mathcal{C}_i(\mu)\mathcal{O}_i\right], \tag{1}$$

where $C_i(\mu)$ are the Wilson coefficients and $\mathcal{O}_i$ are the relevant operators defined as

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A}\,,\\
\mathcal{O}_2 &= (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{c}_\beta b_\beta)_{V-A}\,,\\
\mathcal{O}_{3(5)} &= (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A(V+A)}\,,\\
\mathcal{O}_{4(6)} &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A(V+A)}\,,\\
\mathcal{O}_{7(9)} &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+A(V-A)}\,,\\
\mathcal{O}_{8(10)} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A(V-A)}\,,
\end{aligned}$$

with α, β being the color indices. The explicit expressions of the Wilson coefficients appearing in the above equations can be found in [34]. In the following, we will neglect $V_{ub}V_{us}^*$ and take $V_{cb}V_{cs}^* = -V_{tb}V_{ts}^*$ for simplicity.

We define, in the rest frame of the B^+ meson, p_1, p_2 and p_3 to be the four-momenta of B^+, $X(3872)$ and K^+, and k_1, k_2 and k_3 to be the momenta of the valence quarks inside $B^+(u)$, $X(3872)(c)$ and $K^+(u)$ respectively. Then we parameterize the light cone momenta, with all the light quarks and mesons being treated as massless,

$$\begin{aligned}
p_1 &= \frac{m_B}{\sqrt{2}}(1,1,\mathbf{0}_{\mathrm{T}}) = \left(p_1^+, p_1^-, \mathbf{0}_{\mathrm{T}}\right),\\
p_2 &= \frac{m_B}{\sqrt{2}}(1,r^2,\mathbf{0}_{\mathrm{T}}) = \left(p_2^+, p_2^-, \mathbf{0}_{\mathrm{T}}\right),\\
p_3 &= \frac{m_B}{\sqrt{2}}\left(0,1-r^2,\mathbf{0}_{\mathrm{T}}\right), \qquad k_1 = \left(x_1 p_1^+, 0, \mathbf{k}_{1\mathrm{T}}\right),\\
k_2 &= \left(x_2 p_2^+, x_2 p_2^-, \mathbf{k}_{2\mathrm{T}}\right), \qquad k_3 = \left(0, x_3 p_3^-, \mathbf{k}_{3\mathrm{T}}\right),
\end{aligned} \tag{2}$$

where the mass ratio r is set as $r = m_X/m_B$. x_i are the fractions of the longitudinal momenta of the valence quarks. The superscripts $+$ and $-$ mean that the three-momentum is parallel or anti-parallel to the positive z direction which is defined as the direction of the three-momentum of the $X(3872)$ produced. $\mathbf{k}_{1\mathrm{T}}$, $\mathbf{k}_{2\mathrm{T}}$ and $\mathbf{k}_{3\mathrm{T}}$ are the transverse momenta of the valence quarks inside B^+, $X(3872)$ and K^+, respectively.

The decay width of $B^+ \to X(3872)K^+$ is written as

$$\Gamma = \frac{G_{\mathrm{F}}^2}{2}\frac{|\mathbf{p}_f|}{8\pi m_B^2}|\mathcal{M}|^2, \tag{3}$$

where $|\mathbf{p}_f|$ denotes the three-momentum of the produced meson in the center-of-mass frame of the B^+ meson. In the following, we will calculate the decay amplitude $\mathcal{M}$ in the framework of pQCD.

2.1 Wavefunctions

The B^+ meson light cone wavefunction is usually written as [35–37]

$$\begin{aligned}
&\int_0^1 \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{\mathrm{i}kz}\langle 0|\bar{b}_\alpha(0)u_\beta(z)|B^+(p)\rangle\\
&= -\frac{\mathrm{i}}{\sqrt{2N_c}}\left\{(\not{p} + m_B)\gamma_5\left[\phi_B(\mathbf{k}) - \frac{\not{n} - \not{v}}{\sqrt{2}}\bar{\phi}_B(\mathbf{k})\right]\right\}_{\beta\alpha},
\end{aligned} \tag{4}$$

where $n \equiv (1,0,\mathbf{0}_{\mathrm{T}})$ and $v \equiv (0,1,\mathbf{0}_{\mathrm{T}})$ denote the unit vectors corresponding to the plus and minus directions respectively. In (4), two different Lorentz structures exist in the B^+ meson wavefunctions. $\phi_B(\mathbf{k})$ and $\bar{\phi}_B(\mathbf{k})$ satisfy the following normalization conditions respectively:

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\phi_B(\mathbf{k}) = \frac{f_B}{2\sqrt{2N_c}}\,, \qquad \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\bar{\phi}_B(\mathbf{k}) = 0\,. \tag{5}$$

In the numerical calculation, one usually ignores the contribution of $\bar{\phi}_B(\mathbf{k})$ [38, 39] and only takes the contribution from

$$\Phi_B = \frac{1}{\sqrt{2N_c}}(\not{p} + m_B)\gamma_5\phi_B(\mathbf{k})\,. \tag{6}$$

The twist-3 light cone distribution amplitude of the K meson is expressed as

$$\begin{aligned}
&\langle \bar{K}^+(p)|\bar{u}_\beta(z)s_\alpha(0)|0\rangle\\
&= \frac{\mathrm{i}}{\sqrt{2N_c}}\int_0^1 \mathrm{d}x e^{\mathrm{i}xpz}\{\gamma_5\,\not{p}\phi_K^A(x) + m_0^K\gamma_5\phi_K^P(x)\\
&\quad + m_0^K[\gamma_5(\not{v}\,\not{n} - 1)]\phi_K^{\mathrm{T}}(x)\}_{\beta\alpha}\,,
\end{aligned} \tag{7}$$

where $m_0^K = \frac{m_K^2}{m_s + m_u}$, and x is the momentum fraction carried by the u quark in the K^+ meson. The expressions of $\phi_K^{A(P,T)}$ are given below.

The light cone distribution amplitude of the $X(3872)$ meson is similar to the χ_{c1} meson which has been proposed in [40],

$$\begin{aligned}
&\langle X(3872)(p,\epsilon_{\rm L})|\bar{c}_\alpha(z)c_\beta(0)|0\rangle \\
&= \frac{1}{\sqrt{2N_c}}\int_0^1 {\rm d}u e^{{\rm i}xpz}\{m_X[\gamma_5\ \not\epsilon_{\rm L}]_{\beta\alpha}\phi_X^{\rm L}(x) \\
&\quad + [\gamma_5\ \not\epsilon_{\rm L}\ \not p]_{\beta\alpha}\phi_X^t(x)\}\,, \qquad (8)
\end{aligned}$$

$$\begin{aligned}
&\langle X(3872)(p,\epsilon_{\rm T})|\bar{c}_\alpha(z)c_\beta(0)|0\rangle \\
&= \frac{1}{\sqrt{2N_c}}\int_0^1 {\rm d}u e^{{\rm i}xpz}\{m_X[\gamma_5\ \not\epsilon_{\rm T}]_{\beta\alpha}\phi_X^{V}(x) \\
&\quad + [\gamma_5\ \not\epsilon_{\rm T}\ \not p]_{\beta\alpha}\phi_X^{\rm T}(x)\}\,. \qquad (9)
\end{aligned}$$

Because the $X(3872)$ produced in the transition $B \to X(3872)K$ can only be longitudinally polarized, the distribution amplitude corresponding to transversely polarized $X(3872)$ does not contribute, and we neglect its explicit form in the later text.

2.2 The calculation of the decay amplitude

The quark diagrams which contribute to the transition amplitude of $B^+ \to X(3872) + K^+$ are displayed in Fig. 1.

At first, we can calculate the hard kernels for each diagram displayed in Fig. 1 one by one. As for the factorizable diagrams Fig. 1a and b, we have

$$\begin{aligned}
&H^{(\rm a)}_{\alpha\beta\alpha'\beta'}(p_1,p_2,p_3) \\
&= 4{\rm i}f_X m_X\varepsilon_\mu\bigg[\mathcal{C}_1+\frac{1}{3}\mathcal{C}_2+\mathcal{C}_3+\frac{1}{3}\mathcal{C}_4-\mathcal{C}_5-\frac{1}{3}\mathcal{C}_6 \\
&\quad +\frac{3}{2}e_c\bigg(-\mathcal{C}_7-\frac{1}{3}\mathcal{C}_8+\mathcal{C}_9+\frac{1}{3}\mathcal{C}_{10}\bigg)\bigg] \\
&\quad \times\bigg[{\rm i}g_s\gamma_\nu\frac{\rm i}{-(\not p_1-\not k_3)-m_b}\gamma_\mu(1-\gamma_5)\bigg]_{\alpha\alpha'} \\
&\quad \times({\rm i}g_s\gamma^\nu)_{\beta'\beta}\frac{-{\rm i}}{(k_1-k_3)^2}\,, \qquad (10)
\end{aligned}$$

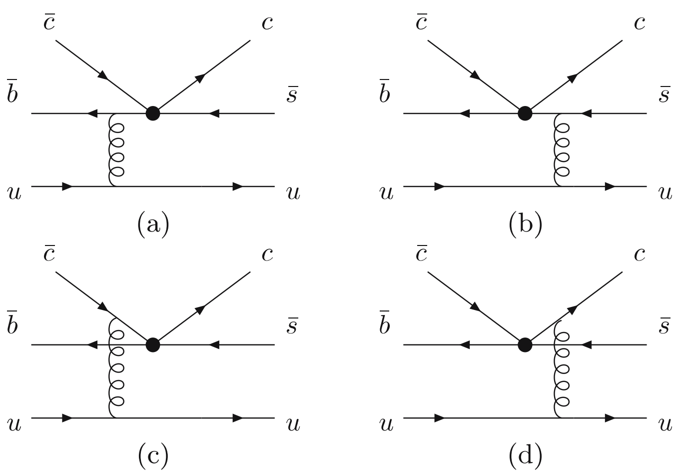

Fig. 1. Feynman diagrams corresponding to the calculation of hard amplitudes in $B^+ \to X(3872)K^+$

$$\begin{aligned}
&H^{(\rm b)}_{\alpha\beta\alpha'\beta'}(p_1,p_2,p_3) \\
&= 4{\rm i}f_X m_X\varepsilon_\mu\bigg[\mathcal{C}_1+\frac{1}{3}\mathcal{C}_2+\mathcal{C}_3+\frac{1}{3}\mathcal{C}_4-\mathcal{C}_5-\frac{1}{3}\mathcal{C}_6 \\
&\quad +\frac{3}{2}e_c\bigg(-\mathcal{C}_7-\frac{1}{3}\mathcal{C}_8+\mathcal{C}_9+\frac{1}{3}\mathcal{C}_{10}\bigg)\bigg] \\
&\quad \times\bigg[\gamma_\mu(1-\gamma_5)\frac{\rm i}{-(\not p_3-\not k_1)}{\rm i}g_s\gamma_\nu\bigg]_{\alpha\alpha'} \\
&\quad \times({\rm i}g_s\gamma^\nu)_{\beta'\beta}\frac{-{\rm i}}{(k_1-k_3)^2}\,. \qquad (11)
\end{aligned}$$

As far as the non-factorizable diagrams Fig. 1c and d are concerned, we will divide the operators appearing in the effective Hamiltonian into two categories according to their chirality, i.e. $(V-A)\otimes(V-A)$ and $(V-A)\otimes(V+A)$, for convenience of computation.

For the type of $(V-A)\otimes(V\mp A)$, the hard kernels of Fig. 1c are respectively written as

$$\begin{aligned}
&H^{(\rm c,1)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q) = 4\bigg(\mathcal{C}_2+\mathcal{C}_4+\frac{3}{2}e_c\mathcal{C}_{10}\bigg) \\
&\quad \times\bigg[\gamma_\mu(1-\gamma_5)\frac{\rm i}{-(\not p_2-\not k_2-\not k_1+\not k_3)-m_c}{\rm i}g_s\gamma_\nu\bigg]_{\alpha\rho'} \\
&\quad \times\big[\gamma^\mu(1-\gamma_5)\big]_{\rho\alpha'}\big[{\rm i}g_s\gamma^\nu\big]_{\beta\beta'}\frac{-{\rm i}}{(k_1-k_3)^2}\,, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&H^{(\rm c,2)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q) = -8\bigg(\mathcal{C}_6+\frac{3}{2}e_c\mathcal{C}_8\bigg) \\
&\quad \times\bigg[(1+\gamma_5)\frac{\rm i}{-(\not p_2-\not k_2-\not k_1+\not k_3)-m_c}{\rm i}g_s\gamma_\nu\bigg]_{\alpha\rho'} \\
&\quad \times[(1-\gamma_5)]_{\rho\alpha'}\big[{\rm i}g_s\gamma^\nu\big]_{\beta\beta'}\frac{-{\rm i}}{(k_1-k_3)^2}\,, \qquad (13)
\end{aligned}$$

where the factor -2 in (13) comes from the Fierz transformation on the $(V-A)\otimes(V+A)$ operators.

Similarly, for Fig. 1d, we have

$$\begin{aligned}
&H^{(\rm d,1)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q) = 4\bigg(\mathcal{C}_2+\mathcal{C}_4+\frac{3}{2}e_c\mathcal{C}_{10}\bigg)[\gamma_\mu(1-\gamma_5)]_{\alpha\rho'} \\
&\quad \times\bigg[{\rm i}g_s\gamma_\nu\frac{\rm i}{(\not k_2-\not k_1+\not k_3)-m_c}\gamma^\mu(1-\gamma_5)\bigg]_{\rho\alpha'} \\
&\quad \times\big[{\rm i}g_s\gamma^\nu\big]_{\beta\beta'}\frac{-{\rm i}}{(k_1-k_3)^2}\,, \\
&H^{(\rm d,2)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q) = -8\bigg(\mathcal{C}_6+\frac{3}{2}e_c\mathcal{C}_8\bigg)[(1+\gamma_5)]_{\alpha\rho'} \\
&\quad \times\bigg[{\rm i}g_s\gamma_\nu\frac{\rm i}{(\not k_2-\not k_1+\not k_3)-m_c}(1-\gamma_5)\bigg]_{\rho\alpha'} \\
&\quad \times\big[ig_s\gamma^\nu\big]_{\beta\beta'}\frac{-{\rm i}}{(k_1-k_3)^2}\,. \qquad (14)
\end{aligned}$$

In the above expressions, the indices $1,2$ denote the contributions from $(V-A)\otimes(V-A)$ and $(V-A)\otimes(V+A)$ operators respectively.

Then we can obtain the decay amplitudes $M^{(\rm a)}$, $M^{(\rm b)}$, $M^{(\rm c)}_{1,2}$, $M^{(\rm d)}_{1,2}$ by virtue of contracting the hard kernels with hadronic wavefunctions,

$$\begin{aligned}M^{(\mathrm{a})} = &\frac{4m_X f_X}{N_c}\int \mathrm{d}x_1 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_3\,\mathrm{d}b_3 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_1+\frac{1}{3}\mathcal{C}_2+\mathcal{C}_3+\frac{1}{3}\mathcal{C}_4-\mathcal{C}_5-\frac{1}{3}\mathcal{C}_6\right.\\ &\left.+\frac{3}{2}e_c\left(-\mathcal{C}_7-\frac{1}{3}\mathcal{C}_8+\mathcal{C}_9+\frac{1}{3}\mathcal{C}_{10}\right)\right]\phi_B(x_1,b_1)\\ &\times\left[\phi_K^A(1-x_3)\mathbb{KA}^{(\mathrm{a})}+\phi_K^P(1-x_3)\mathbb{KP}^{(\mathrm{a})}\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}^{(\mathrm{a})}\right]\\ &\times\alpha_{\mathrm{s}}(t_a)\exp[-S(t_a)]S_t(x_3)\Omega_a(x_1,x_3,b_1,b_3)\,, \quad (15)\end{aligned}$$

$$\begin{aligned}M^{(\mathrm{b})} = &\frac{4m_X f_X}{N_c}\int \mathrm{d}x_1 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_3\,\mathrm{d}b_3 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_1+\frac{1}{3}\mathcal{C}_2+\mathcal{C}_3+\frac{1}{3}\mathcal{C}_4-\mathcal{C}_5-\frac{1}{3}\mathcal{C}_6\right.\\ &\left.+\frac{3}{2}e_c\left(-\mathcal{C}_7-\frac{1}{3}\mathcal{C}_8+\mathcal{C}_9+\frac{1}{3}\mathcal{C}_{10}\right)\right]\phi_B(x_1,b_1)\\ &\times\left[\phi_K^A(1-x_3)\mathbb{KA}^{(\mathrm{b})}+\phi_K^P(1-x_3)\mathbb{KP}^{(\mathrm{b})}\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}^{(\mathrm{b})}\right]\\ &\times\alpha_{\mathrm{s}}(t_b)\exp[-S(t_b)]S_t(x_1)\Omega_b(x_1,x_3,b_1,b_3)\,, \quad (16)\end{aligned}$$

$$\begin{aligned}M_1^{(\mathrm{c})} = &\frac{8}{(2N_c)^{3/2}}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_2\,\mathrm{d}b_2 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_2+\mathcal{C}_4+\frac{3}{2}e_c\mathcal{C}_{10}\right]\phi_B(x_1,b_1)\\ &\times\left\{\phi_X^{\mathrm{L}}(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{1,L}^{(\mathrm{c})}+\phi_K^P(1-x_3)\mathbb{KP}_{1,L}^{(\mathrm{c})}\right.\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{1,L}^{(\mathrm{c})}\right]+\phi_X^t(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{1,t}^{(\mathrm{c})}\right.\\ &\left.\left.+\phi_K^P(1-x_3)\mathbb{KP}_{1,t}^{(\mathrm{c})}-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{1,t}^{(\mathrm{c})}\right]\right\}\\ &\times\alpha_{\mathrm{s}}(t_c)\exp[-S(t_c)]\Omega_c(x_1,x_2,x_3,b_1,b_2)\,, \quad (17)\end{aligned}$$

$$\begin{aligned}M_2^{(\mathrm{c})} = &-\frac{16}{(2N_c)^{3/2}}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_2\,\mathrm{d}b_2 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_6+\frac{3}{2}e_c\mathcal{C}_8\right]\phi_B(x_1,b_1)\\ &\times\left\{\phi_X^{\mathrm{L}}(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{2,L}^{(\mathrm{c})}+\phi_K^P(1-x_3)\mathbb{KP}_{2,L}^{(\mathrm{c})}\right.\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{2,L}^{(\mathrm{c})}\right]+\phi_X^t(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{2,t}^{(\mathrm{c})}\right.\\ &\left.\left.+\phi_K^P(1-x_3)\mathbb{KP}_{2,t}^{(\mathrm{c})}-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{2,t}^{(\mathrm{c})}\right]\right\}\\ &\times\alpha_{\mathrm{s}}(t_c)\exp[-S(t_c)]\Omega_c(x_1,x_2,x_3,b_1,b_2)\,, \quad (18)\end{aligned}$$

$$\begin{aligned}M_1^{(\mathrm{d})} = &\frac{8}{(2N_c)^{3/2}}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_2\,\mathrm{d}b_2 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_2+\mathcal{C}_4+\frac{3}{2}e_c\mathcal{C}_{10}\right]\phi_B(x_1,b_1)\\ &\times\left\{\phi_X^{\mathrm{L}}(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{1,L}^{(\mathrm{d})}+\phi_K^P(1-x_3)\mathbb{KP}_{1,L}^{(\mathrm{d})}\right.\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{1,L}^{(\mathrm{d})}\right]+\phi_X^t(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{1,t}^{(\mathrm{d})}\right.\\ &\left.\left.+\phi_K^P(1-x_3)\mathbb{KP}_{1,t}^{(\mathrm{d})}-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{1,t}^{(\mathrm{d})}\right]\right\}\\ &\times\alpha_{\mathrm{s}}(t_d)\exp[-S(t_d)]\Omega_d(x_1,x_2,x_3,b_1,b_2)\,, \quad (19)\end{aligned}$$

$$\begin{aligned}M_2^{(\mathrm{d})} = &-\frac{16}{(2N_c)^{3/2}}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}x_3 \int b_1\,\mathrm{d}b_1 \int b_2\,\mathrm{d}b_2 \\ &\times \int \mathrm{d}\theta V_{cb}V_{cs}^*\left[\mathcal{C}_6+\frac{3}{2}e_c\mathcal{C}_8\right]\phi_B(x_1,b_1)\\ &\times\left\{\phi_X^{\mathrm{L}}(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{2,L}^{(\mathrm{d})}+\phi_K^P(1-x_3)\mathbb{KP}_{2,L}^{(\mathrm{d})}\right.\right.\\ &\left.-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{2,L}^{(\mathrm{d})}\right]+\phi_X^t(x_2)\left[\phi_K^A(1-x_3)\mathbb{KA}_{2,t}^{(\mathrm{d})}\right.\\ &\left.\left.+\phi_K^P(1-x_3)\mathbb{KP}_{2,t}^{(\mathrm{d})}-\phi_K^{\mathrm{T}}(1-x_3)\mathbb{KT}_{2,t}^{(\mathrm{d})}\right]\right\}\\ &\times\alpha_{\mathrm{s}}(t_d)\exp[-S(t_d)]\Omega_d(x_1,x_2,x_3,b_1,b_2)\,, \quad (20)\end{aligned}$$

where the explicit expressions of $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(\mathrm{a})}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(\mathrm{b})}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(\mathrm{c})}_{1(2),L(t)}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(\mathrm{d})}_{1(2),L(t)}$, $\Omega_i(x_1,x_2,x_3,b_1,b_2,b_3)$ and Sudakov factors of $S(t_i)$, $S_t(x)$ ($i=a$–d) are collected in the appendix.

Finally, the total amplitude can be written as

$$\mathcal{M} = M^{(\mathrm{a})}+M^{(\mathrm{b})}+M_1^{(\mathrm{c})}+M_2^{(\mathrm{c})}+M_1^{(\mathrm{d})}+M_2^{(\mathrm{d})}.$$

3 Numerical results

The wavefunction of the B meson is given by [38, 39]

$$\phi_B(x,b) = \frac{N_B}{2\sqrt{2N_c}}f_B x^2(1-x)^2\exp\left[-\frac{M_B^2x^2}{2\omega_b^2}-\frac{1}{2}(b\omega_b)^2\right], \quad (21)$$

where $\omega_b = 0.4\,\mathrm{GeV}$ and $N_B = 2.4\times10^3$. The decay constant of the B meson is $f_B = 0.19\,\mathrm{GeV}$.

The distribution amplitudes of $X(3872)$ can be derived following the method given in [40, 41]:

$$\begin{aligned}\phi_X^{\mathrm{L}}(x) &= 24.80\frac{f_X}{2\sqrt{2N_c}}x(1-x)\\ &\quad\times\left\{\frac{x(1-x)(1-2x)^2[1-4x(1-x)]}{[1-3.47x(1-x)]^3}\right\}^{0.7},\\ \phi_X^t(x) &= 13.56\frac{f_X}{2\sqrt{2N_c}}(1-2x)^2\\ &\quad\times\left\{\frac{x(1-x)(1-2x)^2[1-4x(1-x)]}{[1-3.47x(1-x)]^3}\right\}^{0.7}, \quad (22)\end{aligned}$$

where the $X(3872)$ decay constant f_X is set to be 0.335 GeV, which is the same as that of χ_{c1}.

In a recent work [42], the K^- meson wavefunction distribution amplitudes used in (7) is given as

$$\begin{aligned}\phi_K^A(x) &= \frac{f_K}{2\sqrt{2N_c}}\left\{6x(1-x)\left(1+a_1^KC_1^{3/2}(2x-1)\right.\right.\\ &\quad\left.\left.+a_2^KC_2^{3/2}(2x-1)\right)\right\}, \quad (23)\\ \phi_K^P(x) &= \frac{f_K}{2\sqrt{2N_c}}\left\{1+3\rho_+^K\left(1+6a_2^K\right)-9\rho_-^Ka_1^K\right.\\ &\quad+C_1^{1/2}(2x-1)\left[\frac{27}{2}\rho_+^Ka_1^K-\rho_-^K\left(\frac{3}{2}+27a_2^K\right)\right]\\ &\quad+C_2^{1/2}(2x-1)\left(30\eta_{3K}+15\rho_+^Ka_2^K-3\rho_-^Ka_1^K\right)\end{aligned}$$

$$
\begin{aligned}
&+ C_3^{1/2}(2x-1)\left(10\eta_{3K}\lambda_{3K} - \frac{9}{2}\rho_-^K a_2^K\right) \\
&- 3\eta_{3K}\omega_{3K} C_4^{1/2}(2x-1) \\
&+ \frac{3}{2}\left(\rho_+^K + \rho_-^K\right)\left(1 - 3a_1^K + 6a_2^K\right)\ln x \\
&+ \frac{3}{2}\left(\rho_+^K - \rho_-^K\right)\left(1 + 3a_1^K + 6a_2^K\right)\ln \bar{x}\Big\}\,, \qquad (24)
\end{aligned}
$$

$$
\phi_K^{\mathrm{T}} = \frac{1}{6}\frac{\mathrm{d}\phi_K^\sigma(x)}{\mathrm{d}x}\,, \qquad (25)
$$

$$
\begin{aligned}
\phi_K^\sigma(x) = \frac{f_K}{2\sqrt{2N_c}}\Big\{&6x\bar{x}\Big[1 + \frac{3}{2}\rho_+^K + 15\rho_+^K a_2^K - \frac{15}{2}\rho_-^K a_1^K \\
&+ \left(3\rho_+^K a_1^K - \frac{15}{2}\rho_-^K a_2^K\right) C_1^{3/2}(2x-1) \\
&+ \left(5\eta_{3K} - \frac{1}{2}\eta_{3K}\omega_{3K} + \frac{3}{2}\rho_+^K a_2^K\right) C_2^{3/2}(2x-1) \\
&+ \eta_{3K}\lambda_{3K} C_3^{3/2}(2x-1)\Big] \\
&+ 9x\bar{x}\left(\rho_+^K + \rho_-^K\right)\left(1 - 3a_1^K + 6a_2^K\right)\ln x \\
&+ 9x\bar{x}\left(\rho_+^K - \rho_-^K\right)\left(1 + 3a_1^K + 6a_2^K\right)\ln \bar{x}\Big\}\,, \qquad (26)
\end{aligned}
$$

with

$$
\begin{aligned}
\rho_+^K &= \frac{(m_s + m_u)^2}{m_K^2}\,, & \rho_-^K &= \frac{m_s^2 - m_u^2}{m_K^2}\,, \\
\eta_{3K} &= \frac{f_{3K}}{f_K}\frac{m_u + m_s}{m_K^2}\,, & C_1^{1/2}(t) &= t\,, \\
C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1)\,, & C_3^{1/2}(t) &= \frac{1}{2}(5t^3 - t)\,, \\
C_4^{1/2}(t) &= \frac{1}{8}(3 - 30t^2 + 35t^4)\,, & C_1^{3/2}(t) &= 3t\,, \\
C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1)\,, & C_3^{3/2} &= \frac{1}{2}(35t^3 + 3t)\,,
\end{aligned}
$$

where $a_1^K = 0.06 \pm 0.03$, $a_2^K = 0.25 \pm 0.15$, $f_{3K} = (0.45 \pm 0.15) \times 10^{-2}\,\mathrm{GeV}^2$, $\omega_{3K} = -1.2 \pm 0.7$, $\lambda_{3K} = 1.6 \pm 0.4$, $f_K = 0.16\,\mathrm{GeV}$, $m_s = 137 \pm 27\,\mathrm{MeV}$, $m_u = 5.6 \pm 1.6\,\mathrm{MeV}$. It is needed to indicate that x is the momentum fraction carried by the s quark in the K^- meson. To convert the K^- distribution amplitude to that for the K^+ meson, one has to replace $\phi_K^{A,P,T}(x)$ with $\phi_K^{A,P}(1-x)$ and $-\phi_K^{\mathrm{T}}(1-x)$.

Other input parameters used in the text are given here: $m_B = 5.279\,\mathrm{GeV}$, $m_K = 0.494\,\mathrm{GeV}$, $m_c = 1.4\,\mathrm{GeV}$ [43]; $m_{X(3872)} = 3.872\,\mathrm{GeV}$ [1]. For the CKM mixing parameters, we take $s_{12} = 0.2243$, $s_{23} = 0.0413$, $s_{13} = 0.0037$ and $\delta_{13} = 1.05$ [43].

Using the above parameters, finally one obtains the branching ratio of $B^+ \to X(3872)K^+$ (in the numerical calculation, we take the central values listed in the data book for the input parameters):

$$
\mathrm{BR}(B^+ \to X(3872)K^+) = 7.88 \times 10^{-4}.
$$

There are some theoretical uncertainties in our calculations, such as the choice of the factorizable scale, hard scale and non-perturbative parameters in the hadronic wavefunctions. As suggested in [44], the decay rate is more dependent on the choice of the hard scale. So we investigate the effect of the hard scale on $\mathrm{BR}(B^+ \to X(3872)K^+)$ by varying the hard scale as

$$
\begin{aligned}
&\max\left(0.75\sqrt{A_a}, 0.75\sqrt{B_a}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right) \le t_a \\
&\le \max\left(1.25\sqrt{A_a}, 1.25\sqrt{B_a}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right)\,, \\
&\max\left(0.75\sqrt{A_b}, 0.75\sqrt{B_b}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right) \le t_b \\
&\le \max\left(1.25\sqrt{A_b}, 1.25\sqrt{B_b}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right)\,, \\
&\max\left(0.75\sqrt{A_c}, 0.75\sqrt{B_c}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right) \le t_c \\
&\le \max\left(1.25\sqrt{A_c}, 1.25\sqrt{B_c}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right)\,, \\
&\max\left(0.75\sqrt{A_d}, 0.75\sqrt{B_d}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right) \le t_d \\
&\le \max\left(1.25\sqrt{A_d}, 1.25\sqrt{B_d}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right)\,.
\end{aligned}
$$

Then we can obtain

$$
\mathrm{BR}(B^+ \to X(3872)K^+) = 7.88^{+4.87}_{-3.76} \times 10^{-4}.
$$

4 Discussion and conclusion

Though the D0, CDF and BABAR Collaboration one after the other confirmed the existence of $X(3872)$ firstly observed by BELLE, there still exists controversy on its structure. The theoretical explanations mainly include charmonium [6–8] and a molecular state [11–15]. For clarifying this mess, one should study $X(3872)$ in different ways.

B non-leptonic decay is a perfect place to study the production of charmonium. Thus we assume $X(3872)$ to be a regular $2\,^3P_1$ charmonium state, and we obtain the branching ratio of $B^+ \to X(3872)K^+$ in the pQCD approach. Our calculation indicates that the order of magnitude of $B^+ \to X(3872)K^+$ is $7.88^{+4.87}_{-3.76} \times 10^{-4}$, which is larger than that given in [9]. Probably this difference can be induced by the difference between the pQCD approach and the model used in [9]. At present, the experimental results only indicate $\mathrm{BR}(B^+ \to X(3872)K^+) < 3.2 \times 10^{-4}$ [26, 27]. Obviously our numerical result on $B^+ \to X(3872) + K^+$ is larger than the current upper bound set by BABAR within the error bar. If both our calculation in the pQCD approach and the experimental results are reliable, a pure charmonium assignment for $X(3872)$ is not suitable.

Experiments gave the branching ratio of the decay chains relevant to $X(3872)$ [1–4]:

$$
\begin{aligned}
&\mathrm{BR}(B^+ \to X(3872)K^+)\mathrm{BR}(X(3872) \to J/\psi\pi^+\pi^-) \\
&\quad = (1.3 \pm 0.3) \times 10^{-5}.
\end{aligned}
$$

Using our numerical results of $B^+ \to X(3872) + K^+$ calculated by the pQCD approach assuming $X(3872)$ as $2\,^3P_1$ charmonium, we can obtain $\mathrm{BR}(X(3872) \to J/\psi\pi^+\pi^-) = (0.8\text{–}3)\%$. Very recently, BELLE reported a new decay channel of $X(3872)$ [5]:

$$
\begin{aligned}
&\mathrm{BR}(B^+ \to X(3872)K^+)\mathrm{BR}(X(3872) \to D^0\bar{D}^0\pi^0) \\
&\quad = \left(1.27 \pm 0.31^{+0.22}_{-0.39}\right) \times 10^{-4}.
\end{aligned}
$$

Then we obtain $\mathrm{BR}(X(3872) \to D^0\bar{D}^0\pi^0) = (5\text{–}44)\%$.

The above discussions on $X(3872)$ are based on supposing $X(3872)$ to be a pure charmonium state. Thus we urge our experimental colleagues to measure these decay channels. It will help us to finally determine the constituents of $X(3872)$.

As a short summary, due to the absence of accurate experimental information on $B \to X(3872)+K$ and the decays of $X(3872)$, one cannot make a definite conclusion about the structure of $X(3872)$. A more decisive conclusion should be made as more accurate data are accumulated by future experiments such as BELLE, BABAR and future LHCb.

Appendix: Some relevant functions appearing in the text

The explicit expressions of $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(a)}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(b)}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(c)}_{1(2),L(t)}$, $\mathbb{KA}(\mathbb{P},\mathbb{T})^{(d)}_{1(2),L(t)}$ which come from the contraction of hard kernel and hadronic wavefunctions are given here:

$$\mathbb{KA}^{(a)} = -\frac{4m_b^3(-1+r^2)\left[-1+x_3(-1+r^2)\right]}{r}, \tag{A.1}$$

$$\mathbb{KP}^{(a)} = -\frac{4m_b^2 m_0^K(-1+r^2)(-1+2x_3)}{r}, \tag{A.2}$$

$$\mathbb{KT}^{(a)} = -\frac{4m_b^2 m_0^K(1+r^2-2x_3)}{r}, \tag{A.3}$$

$$\mathbb{KA}^{(b)} = -4m_b^3 x_1 r(-1+r^2), \tag{A.4}$$

$$\mathbb{KP}^{(b)} = -\frac{8m_b^2 m_0^K\left[1+r^2(-1+x_1)\right]}{r}, \tag{A.5}$$

$$\mathbb{KA}^{(c)}_{1,L} = -\frac{16m_b^3 m_X(-1+r^2)^2(-1+x_1+x_2)}{r}, \tag{A.6}$$

$$\mathbb{KA}^{(c)}_{1,t} = 16m_b^3 m_c r(-1+r^2), \tag{A.7}$$

$$\mathbb{KP}^{(c)}_{1,L} = -\frac{16m_b^2 m_0^K m_X\left[r^2(x_1-x_3)+x_3\right]}{r}, \tag{A.8}$$

$$\mathbb{KT}^{(c)}_{1,L} = -\frac{16m_b^2 m_0^K m_X\left[r^4(-1+x_1+x_2)+x_3-r^2(-1+x_2+x_3)\right]}{r(-1+r^2)}, \tag{A.9}$$

$$\mathbb{KA}^{(c)}_{2,L} = \frac{8m_b^3 m_X(-1+r^2)\left[-1+x_1+x_2-x_3+r^2(-1+x_2+x_3)\right]}{r}, \tag{A.10}$$

$$\mathbb{KA}^{(c)}_{2,t} = -8m_b^3 m_c(-1+r^2)r, \tag{A.11}$$

$$\mathbb{KP}^{(c)}_{2,L} = -\frac{8m_b^2 m_0^K m_X\left[r^2(x_1-x_3)+x_3\right]}{r}, \tag{A.12}$$

$$\mathbb{KT}^{(c)}_{2,L} = \frac{8m_b^2 m_0^K m_X\left[r^4(-1+x_1+x_2)+x_3-r^2(-1+x_2+x_3)\right]}{r(-1+r^2)}, \tag{A.13}$$

$$\mathbb{KT}^{(c)}_{2,t} = -16m_b^2 m_0^K m_c r \tag{A.14}$$

$$\mathbb{KA}^{(d)}_{1,L} = -\frac{16m_b^3 m_X(-1+r^2)\left[x_1-x_2-r^2(x_2-x_3)-x_3\right]}{r}, \tag{A.15}$$

$$\mathbb{KA}^{(d)}_{1,t} = -16m_b^3 m_c(-1+r^2)r, \tag{A.16}$$

$$\mathbb{KP}^{(d)}_{1,L} = \frac{16m_b^2 m_0^K m_X\left[r^2(x_1-x_3)+x_3\right]}{r}, \tag{A.17}$$

$$\mathbb{KT}^{(d)}_{1,L} = -\frac{16m_b^2 m_0^K m_X\left[r^4(x_1-x_2)+r^2(x_2-x_3)+x_3\right]}{r(-1+r^2)}, \tag{A.18}$$

$$\mathbb{KT}^{(d)}_{1,t} = -32m_b^2 m_0^K m_c r, \tag{A.19}$$

$$\mathbb{KA}^{(d)}_{2,L} = \frac{8m_b^3 m_X(-1+r^2)^2(x_1-x_2)}{r}, \tag{A.20}$$

$$\mathbb{KA}^{(d)}_{2,t} = 8m_b^3 m_c(-1+r^2)r, \tag{A.21}$$

$$\mathbb{KP}^{(d)}_{2,L} = \frac{8m_b^2 m_0^K m_X\left[r^2(x_1-x_3)+x_3\right]}{r}, \tag{A.22}$$

$$\mathbb{KT}^{(d)}_{2,L} = \frac{8m_b^2 m_0^K m_X\left[r^4(x_1-x_2)+r^2(x_2-x_3)+x_3\right]}{r(-1+r^2)}, \tag{A.23}$$

$$\begin{aligned}\mathbb{KT}^{(b)} &= \mathbb{KP}^{(c)}_{1,t} = \mathbb{KT}^{(c)}_{1,t} = \mathbb{KP}^{(c)}_{2,t}\\ &= \mathbb{KP}^{(d)}_{1,t} = \mathbb{KP}^{(d)}_{2,t} = \mathbb{KT}^{(d)}_{2,t} = 0.\end{aligned} \tag{A.24}$$

The explicit forms of $\Omega_i(x_1,x_2,x_3,b_1,b_2,b_3)$ $(i=a\text{–}d)$ which come from Fourier transformation to products of propagators corresponding to the quark and the gluon are listed as follows:

$$\Omega_a(x_1,x_3,b_1,b_3) = K_0\left(\sqrt{\mathcal{A}_a}|\mathbf{b}_1+\mathbf{b}_3|\right)K_0\left(\sqrt{\mathcal{B}_a}|\mathbf{b}_1|\right), \tag{A.25}$$

$$\Omega_b(x_1,x_3,b_1,b_3) = K_0\left(\sqrt{\mathcal{A}_b}|\mathbf{b}_1+\mathbf{b}_3|\right)K_0\left(\sqrt{\mathcal{B}_b}|\mathbf{b}_1|\right), \tag{A.26}$$

$$\begin{aligned}\Omega_c(x_1,x_2,x_3,b_1,b_2) = &\Big\{K_0\left(\sqrt{\mathcal{A}_c}|\mathbf{b}_2|\right)\theta(\mathcal{A}_c)\\ &+\frac{\pi}{2}\left[-N_0\left(\sqrt{\mathcal{A}_c}|\mathbf{b}_2|\right)\right.\\ &\left.+\mathrm{i}J_0\left(\sqrt{\mathcal{A}_c}|\mathbf{b}_2|\right)\right]\theta(-\mathcal{A}_c)\Big\}\\ &\times K_0\left(\sqrt{\mathcal{B}_c}|\mathbf{b}_1-\mathbf{b}_2|\right),\end{aligned} \tag{A.27}$$

$$\begin{aligned}\Omega_d(x_1,x_2,x_3,b_1,b_2) = &\Big\{K_0\left(\sqrt{\mathcal{A}_d}|\mathbf{b}_2|\right)\theta(\mathcal{A}_d)\\ &+\frac{\pi}{2}\left[-N_0\left(\sqrt{\mathcal{A}_d}|\mathbf{b}_2|\right)\right.\\ &\left.+\mathrm{i}J_0\left(\sqrt{\mathcal{A}_d}|\mathbf{b}_2|\right)\right]\theta(-\mathcal{A}_d)\Big\}\\ &\times K_0\left(\sqrt{\mathcal{B}_d}|\mathbf{b}_1-\mathbf{b}_2|\right),\end{aligned} \tag{A.28}$$

with

$$\begin{aligned}&\mathcal{A}_a = x_3(1-r^2)m_b^2, \qquad \mathcal{B}_a = x_1x_3(1-r^2)m_b^2,\\ &\mathcal{A}_b = x_1(1-r^2)m_b^2, \qquad \mathcal{B}_b = x_1x_3(1-r^2)m_b^2,\\ &\mathcal{A}_c = m_c^2-(1-x_1-x_2)\left[(1-x_2)r^2+x_3(1-r^2)\right]m_b^2,\\ &\mathcal{B}_c = x_1x_3(1-r^2)m_b^2,\\ &\mathcal{A}_d = m_c^2-(x_2-x_1)\left[x_2r^2+x_3(1-r^2)\right]m_b^2,\\ &\mathcal{B}_d = x_1x_3(1-r^2)m_b^2.\end{aligned}$$

Here J_i, N_i are the ith order Bessel functions of the first and second kind respectively, and K_i denotes the ith order modified Bessel functions.

The explicit expressions for the Sudakov factor coming from the resummation of the double logarithm appearing in high order radiative corrections to the diagrams are also given below:

$$\begin{aligned} S(t_a) &= s\left(x_1 p_1^+, b_1\right) + s\left(x_3 p_3^-, b_3\right) + s\left((1-x_3)p_3^-, b_3\right) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_a/\Lambda_{\rm QCD})}{\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_a/\Lambda_{\rm QCD})}{\ln(b_3\Lambda_{\rm QCD})}\right], \end{aligned} \tag{A.29}$$

$$\begin{aligned} S(t_b) &= s\left(x_1 p_1^+, b_1\right) + s\left(x_3 p_3^-, b_3\right) + s\left((1-x_3)p_3^-, b_3\right) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_b/\Lambda_{\rm QCD})}{\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_b/\Lambda_{\rm QCD})}{\ln(b_3\Lambda_{\rm QCD})}\right], \end{aligned} \tag{A.30}$$

$$\begin{aligned} S(t_c) &= s\left(x_1 p_1^+, b_1\right) + s\left(x_3 p_3^-, b_3\right) + s\left((1-x_3)p_3^-, b_3\right) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{-\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{\ln(b_2\Lambda_{\rm QCD})}\right. \\ &\quad \left. + \ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{\ln(b_3\Lambda_{\rm QCD})}\right], \end{aligned} \tag{A.31}$$

$$\begin{aligned} S(t_d) &= s\left(x_1 p_1^+, b_1\right) + s\left(x_3 p_3^-, b_3\right) + s\left((1-x_3)p_3^-, b_3\right) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{-\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{\ln(b_2\Lambda_{\rm QCD})}\right. \\ &\quad \left. + \ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{\ln(b_3\Lambda_{\rm QCD})}\right], \end{aligned} \tag{A.32}$$

where

$$\begin{aligned} t_a &= \max\left(\sqrt{A_a}, \sqrt{B_a}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right), \\ t_b &= \max\left(\sqrt{A_b}, \sqrt{B_b}, 1/b_1, 1/b_3, 1/|\mathbf{b}_1 + \mathbf{b}_3|\right), \\ t_c &= \max\left(\sqrt{A_c}, \sqrt{B_c}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right), \\ t_d &= \max\left(\sqrt{A_d}, \sqrt{B_d}, 1/b_1, 1/b_2, 1/b_3, 1/|\mathbf{b}_1 - \mathbf{b}_2|\right). \end{aligned}$$

The explicit expressions for the Sudakov factors are given in [45–47] as

$$\begin{aligned} S_t(x) &= \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \\ s(\omega, Q) &= \int_\omega^Q \frac{\mathrm{d}p}{p}\left[\ln\left(\frac{Q}{p}\right)A[\alpha_s(p)] + B[\alpha_s(p)]\right], \\ A &= C_F\frac{\alpha_s}{\pi} \\ &\quad + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{7}n_f + \frac{8}{3}\beta_0\ln\left(\frac{e^{\gamma_E}}{2}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2, \\ B &= \frac{2}{3}\frac{\alpha_s}{\pi}\ln\left(\frac{e^{2\gamma_E - 1}}{2}\right), \\ \gamma_q(\alpha_s(\mu)) &= -\alpha_s(\mu)/\pi, \\ \beta_0 &= \frac{33-2n_f}{2}, \end{aligned}$$

where c is set as 0.4 in our work, and γ_E is the Euler constant. n_f is the flavor number, and γ_q is the anomalous dimension. We will take n_f equal to 4 in our numerical calculations.

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